SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2016
HIGHER SCHOOL CERTIFICATE
ASSESSMENT TASK #2

Mathematics

General Instructions

- Reading Time – 5 Minutes
- Working time – 90 Minutes
- Write using black or blue pen. Pencil may be used for diagrams.
- Board approved calculators maybe used.
- All necessary working should be shown in every question.
- All answers to be given in simplified exact form, unless otherwise stated.
- Hand in your answers in 4 separate bundles. Multiple Choice (Q1-5), Question 6, Question 7 and Question 8.

Total Marks – 70

- Attempt questions 1-8
- All questions are **NOT** of equal value.

Examiner: *A Ward*

Section A – Answer on Multiple Choice Answer Paper.

5 Marks

1. Given 1

$$f(x)=\begin{cases}-5 \text{ for } x\le -3\\ 2x \text{ for } -3<x<0\\ x^2 \text{ for } x\ge 0\end{cases}$$

Find the value of $f(-3)\div f(3)$

A. $x=\dfrac{-5}{9}$

B. $x=\dfrac{2}{3}$

C. $x=\dfrac{1}{2}$

D. $x=\dfrac{-5}{6}$

2. Convert $\dfrac{3\pi}{5}$ radians to degrees. 1

A. 108^{o}

B. 54^{o}

C. 216^{o}

D. 540^{o}

3. Find the primitive of: $e^{7x}+14$ 1

A. $7e^{7x}+14+c$

B. $\dfrac{e^{7x}}{7}+14x+c$

C. $e^{7x}+14x+c$

D. $7e^{7x}+14x+c$

4. Differentiate: $\log_e(4x+3)$ 1

A. $4\log_e(4x+3)$

B. $\dfrac{4}{\log_e(4x+3)}$

C. $\dfrac{4}{4x+3}$

D. $\dfrac{4x+3}{4}$

5. What is the exact value of $\cos\dfrac{7\pi}{6}$? 1

A. $\dfrac{\sqrt{3}}{2}$

B. $\dfrac{-\sqrt{3}}{2}$

C. $\dfrac{1}{2}$

D. $\dfrac{-1}{2}$

End of Multiple Choice

Question 6 Overleaf

Question 6 – Start a new booklet.

20 Marks

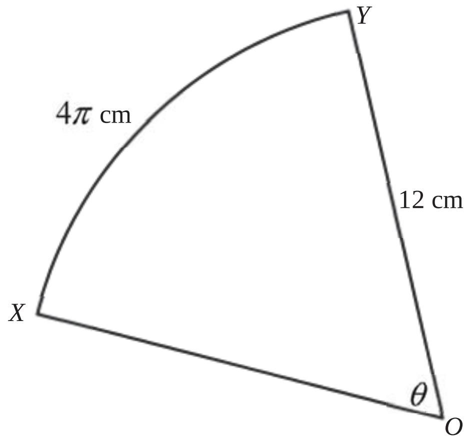
a. Find:

$$\int \frac{3}{2x+6}\,dx$$ **2**

b. Draw on a number line the solution of:

$$|2x-1| \geq 5$$ **2**

c.



In the diagram, *XY* is an arc of a circle with centre *O* and radius 12 cm. The length of the arc *XY* is 4π cm.

i. Find the exact size of θ in radians. **1**

ii. Find the exact area of sector *OXY*. **1**

d. The co-ordinates of the points *A*, *B* and *C* are $(-4,3)$, $(0,5)$ and $(9,2)$ respectively. (Hint: draw a diagram)

i. Find the length of the interval *BC*. **1**

ii. Show that the equation of the line *l*, drawn through *A* parallel to *BC* is $x+3y-5=0$ **2**

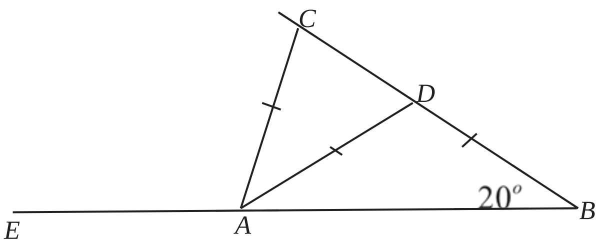
iii. Find the co-ordinates of *D*, the point where the line *l* meets the *x*-axis. **1**

iv. Prove *ABCD* is a parallelogram. **2**

v. Find the perpendicular distance from the point *B* to line *l*. **2**

vi. Hence, or otherwise, find the area of the parallelogram *ABCD*. **1**

e.



In the diagram $CA = AD = DB$ and $\angle EBD = 20°$

i. Copy this diagram onto your answer sheet.

ii. Show $\angle ADC = 40°$, giving reasons. **2**

iii. Hence, find the size $\angle CAE$ giving reasons **1**

f. A point P moves so that it is always equidistant from the points $A(-4, 0)$ and $B(4, 0)$. Find the locus of P. **2**

End of Question 6.

Question 7 Overleaf

Question 7 – Start a new booklet.

23 Marks

a. Differentiate:

i. $3xe^{5x}$ **2**

ii. $\ln\left(\dfrac{x+2}{x-2}\right)$ **2**

b. The table shows the values of a function $f(x)$ for five values of x.

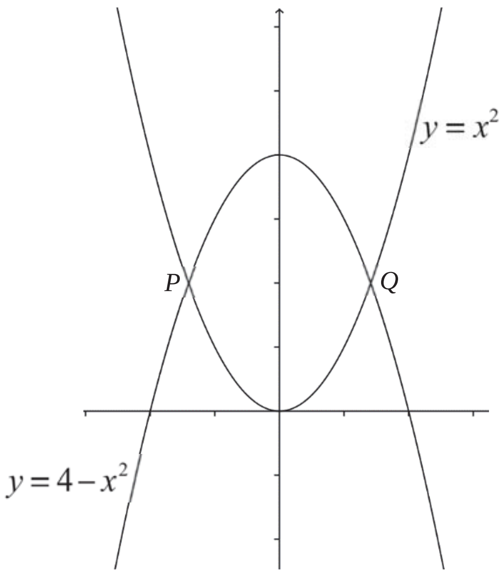
x	1	1.5	2	2.5	3
$f(x)$	7	3	-1	5	9

Use the Trapezoidal rule with these five values to find the value of:

$$\int_1^3 f(x)dx$$

3

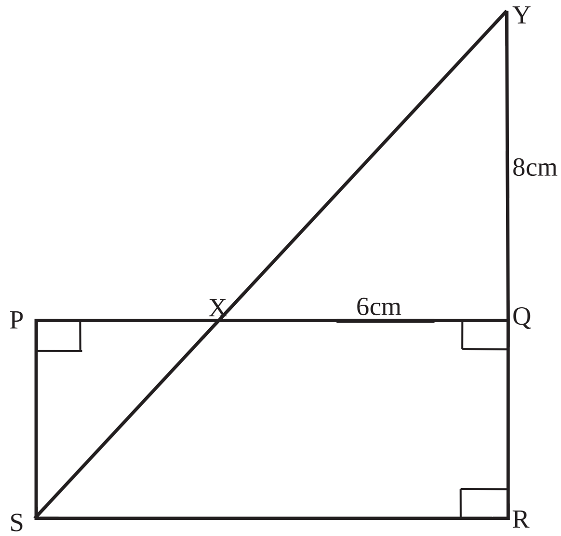
c. The curves $y = x^2$ and $y = 4 - x^2$ are shown below. The two curves intersect at P and Q.



i. Write down the co-ordinates of P and Q. **2**

ii. Hence, find the exact area of the region enclosed by $y = x^2$ and $y = 4 - x^2$ **3**

d.



In the diagram, *PQRS* is a rectangle and *SR*=3*PS*. *R*,*Q* and *Y* are collinear points. *XQ* = 6cm and *YQ* = 8cm.

i. Prove that $\Delta YQX \parallel\!| \Delta YRS$ **2**

ii. Hence find the length of *PS*. **1**

e. Graph the curves then shade the intersection of the regions defined by :

$$2y \geq x^2 - 5 \text{ and } y < x\text{-}1$$ **3**

f. The region bounded by the curve $y = e^x + e^{-x}$, the x axis and the lines $x = 0$ and $x = 2$ is rotated around the x axis. Find the volume of the solid formed. (Leave your answer in terms of e). **3**

g. A curve has gradient function $\frac{dy}{dx} = e^{3x}$. Find the equation of the curve if it passes through the point $(0, 2)$ **2**

End of Question 7

Question 8 Overleaf

Question 8 – Start a new booklet.

22 Marks

a. If $f(x)=x+\dfrac{1}{x}$

i. Solve $f(x)=-2$. **2**

ii. Show whether the function is odd, even or neither. **1**

iii. Write down the domain and range of $f(x)$. **2**

b. Use Simpson's rule with 5 function values to find the approximate volume, to 2 decimal places, when the area bounded by the curve

$$y=\frac{1}{\sqrt{4+x^2}}$$

, x-axis and the lines $x=1$ and $x=5$, is rotated about the x-axis **3**

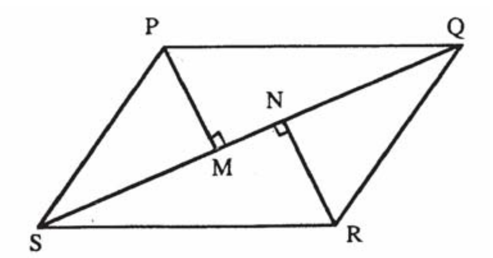
c. Evaluate $\displaystyle\int_1^{e^3}\frac{7}{x}\,dx$. **2**

d. i. Draw the graphs of $y=3\cos x$ and $y=1-x$ on the same axes for $-2\pi\le x\le 2\pi$ **2**

ii. Explain why all the solutions of the equation $3\cos x=1-x$ must lie between $x=-2$ and $x=4$ **1**

e. Find the equation of the circle which is concentric to circle $x^2+y^2+8x+2y+8=0$ and which passes through the point $(1,7)$ **3**

f.



PQRS is a parallelogram. *PM* and *RN* are perpendicular to *QS*.

i. Copy the diagram into your answer booklet.

ii. Prove the *PNRM* a parallelogram. **3**

g. The function represented by the equation $x = 3\sin(nt) + 6$ has a period equal to $\dfrac{3\pi}{4}$. Determine the value of n. **1**

h. Graph the piecemeal function $v(t) = \begin{cases} Ae^{kt} & 0 \le t < \dfrac{1}{k} \\ Ae & t \ge \dfrac{1}{k} \end{cases}$

where A, $k > 1$ are constants. **2**

End of Question 8

End of Examination

SYDNEY BOYS HIGH SCHOOL
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2016

HSC Task #2

Mathematics 2U

Suggested Solutions & Markers' Comments

QUESTION	Marker
1 – 5	–
6	PB
7	JWC
8	RB

Multiple Choice Answers

1. A
2. A
3. B
4. C
5. B

Marks

1. Given 1

$$f(x)=\begin{cases}-5 \text{ for } x\le -3\\ 2x \text{ for } -3<x<0\\ x^2 \text{ for } x\ge 0\end{cases}$$

Find the value of $f(-3)\div f(3)$

A. $x=\dfrac{-5}{9}$

B. $x=\dfrac{2}{3}$

C. $x=\dfrac{1}{2}$

D. $x=\dfrac{-5}{6}$

$\dfrac{-5}{9}$

1 A
2 A
3 B
4 C
5 B.

2. Convert $\dfrac{3\pi}{5}$ radians to degrees. 1

A. 108°

B. 54°

C. 216°

D. 540°

$\dfrac{3\times 180}{5} = 108^\circ$

3. Find the primitive of: $e^{7x}+14$ 1

A. $7e^{7x}+14+c$

B. $\dfrac{e^{7x}}{7}+14x+c$

C. $e^{7x}+14x+c$

D. $7e^{7x}+14x+c$

$\int\left(e^{7x}+14\right)dx$

$=\frac{1}{7}e^{7x}+14x+C$

Differentiate ✓

Marks

4. ~~Derive:~~ $\log_e(4x+3)$ 1

A. $4\log_e(4x+3)$

B. $\dfrac{4}{\log_e(4x+3)}$

C. $\dfrac{4}{4x+3}$

D. $\dfrac{4x+3}{4}$

5. What is the exact value of $\cos\dfrac{7\pi}{6}$: 1

$$\frac{7 \times 180}{6} = 210^\circ$$

$$\cos 210^\circ$$

$$= -\cos 30^\circ$$

S | A

T | C

A. $\dfrac{\sqrt{3}}{2}$

B. $\dfrac{-\sqrt{3}}{2}$

C. $\dfrac{1}{2}$

D. $\dfrac{-1}{2}$

End of Multiple Choice

QUESTION 6 (2U)

a. $\displaystyle\int \frac{3}{2x+6}\,dx = \frac{3}{2}\int \frac{dx}{x+3}$

$= \frac{3}{2}\ln|x+3| + c.$

COMMENT alternatively $\frac{3}{2}\ln|2x+6| + c_1$

(answers differ by a constant ie $c \neq c_1$)

~~b~~ (i) $l = r\theta$

$\therefore 4\pi = 12\theta$

$\boxed{\theta = \frac{\pi}{3}}$

(ii) $A = \frac{1}{2}r^2\theta$

$= \frac{1}{2} \times 144 \times \frac{\pi}{3}$

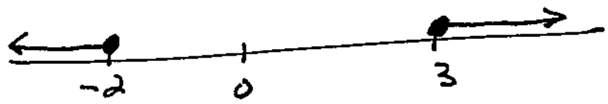
$\therefore \boxed{A = 24\pi \text{ cm}^2}$

COMMENT most students obtained full marks.

b $|2x-1| \geq 5$

$\therefore 2x-1 \geq 5 \quad , \quad 2x-1 \leq -5$

$x \geq 3 \quad , \quad x \leq -2$



COMMENT well done.

(d)

(i) $BC = \sqrt{(5-2)^2+(0-9)^2}$

$= \sqrt{9+81}$

$= \sqrt{90}$

$\boxed{= 3\sqrt{10}}$ units.

(ii) $\frac{y-3}{x+4} = \frac{2-5}{9-0}$

$\frac{y-3}{x+4} = \frac{-1}{3}$

$3y-9 = -x-4$

$\boxed{x+3y-5 = 0}$

(iii) $\boxed{D(5,0)}$

(iv) $m_{AB} = \frac{5-3}{0+4}$

$= \frac{1}{2}$

$m_{CD} = \frac{2-0}{9-5}$

$= \frac{1}{2}$

$\therefore AB \parallel CD$ & $BC \parallel AD$ (data)

$\therefore ABCD$ is a parallelogram.

(v) $d = \left| \frac{1\times 0 + 3\times 5 - 5}{\sqrt{10}} \right|$

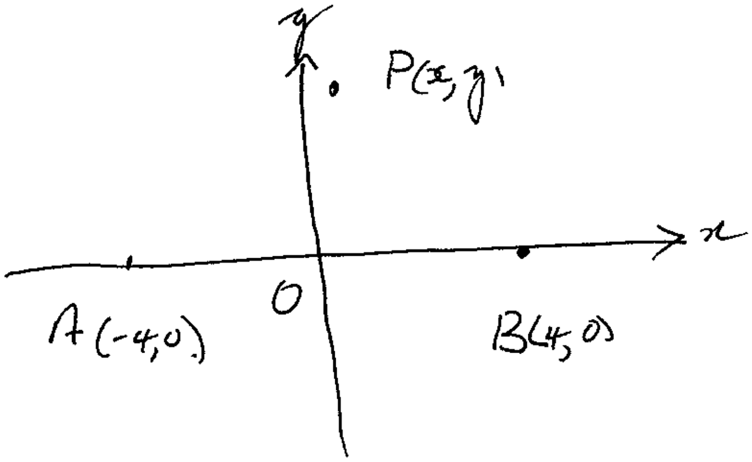
$= \frac{10}{\sqrt{10}}$

$\boxed{= \sqrt{10}.}$

(vi) Area $= 3\sqrt{10} \times \sqrt{10}$

$\boxed{= 30\ u^2}$

f



now $PA = PB$.

ie $\sqrt{(x+4)^2 + (y-0)^2} = \sqrt{(x-4)^2 + (y-0)^2}$.

$$(x+4)^2 + y^2 = (x-4)^2 + y^2$$

$$x^2 + 8x + 16 = x^2 - 8x + 16.$$

$$16x = 0$$

$$\therefore \quad \boxed{x = 0} \quad \text{ie Y-axis}$$

COMMENTS The locus is clearly the perpendicular bisector of the interval AB. ie the y-axis.

NB The instructions clearly state that necessary working be shown

AND

Answers to be given in simplified form ie not $16x=0$ for example rather $x=0$ is simpler.

COMMENTS

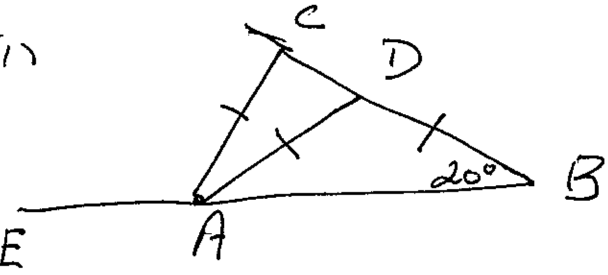
* A number of students had trouble with (ii). They were able to establish correct gradient but didn't involve the point A.

*. (iv) could be done in a variety of ways. eg one pair of sides equal and parallel.

OR diagonals bisect one another.

*. Overall, quite well done.

(e) (i)



(ii) $\angle DAC = 20°$ (base angles of an isosceles triangle)

$\therefore \boxed{\angle ADC = 40°}$ (exterior angle equal to the sum of the interior opposite angles).

(iii) $\angle ACD = 40°$ (base angles of an isosceles triangle)

$\therefore \angle CAE = 40° + 20°$

$\boxed{= 60°}$ (exterior angle equal to the sum of the interior opposite angles)

COMMENT most students obtained full marks.

Year 12 (2 Unit) Half Yearly Examination

Question 7

a i) $y = 3xe^{5x}$

$y' = 3x(5e^{5x}) + e^{5x}(3)$ ✓

$= 15xe^{5x} + 3e^{5x}$

$y' = 3e^{5x}(5x+1)$ ✓ (2)

Generally well-done. Careless mistakes with the differential of e^{5x}

ii) $y = \ln\left(\frac{x+2}{x-2}\right)$

ie $y = \ln(x+2) - \ln(x-2)$

$= \frac{1}{x+2} - \frac{1}{x-2}$

$= \frac{x-2-(x+2)}{(x+2)(x-2)}$ ✓

$\therefore y' = \frac{-4}{(x+2)(x-2)}$ ✓ (2)

The use of log law or chain rule is suitable. However, $ln\frac{x+2}{x-2}$ is not the same as $\frac{ln(x+2)}{ln(x-2)}$

b) $\approx \frac{0.5}{2}(7+2(3+-1+5)+9)$ ✓

$= \frac{0.5}{2} \times 30$ ✓ (3)

$= 7.5$ ✓

Award 1 for the height = 0.5

Award 1 for the value 30

Award for the correct use of the Trapezoidal rule

Some candidates were confused over the Trapezoidal rule.

c i) $4 - x^2 = x^2$

$0 = 2x^2 - 4$

$0 = 2(x+\sqrt{2})(x-\sqrt{2})$

$\therefore P(-\sqrt{2}, 2)$ ✓ (2)

$Q(\sqrt{2}, 2)$ ✓

(−1) with no working

Students were told about the mistake in the question during the examination. Using $y = 2 - x^2$ will make the question significantly easier.

ii) $\int_{-\sqrt{2}}^{\sqrt{2}} (4-x^2) - x^2 \, dx$

$\int_{-\sqrt{2}}^{\sqrt{2}} 4 - 2x^2 \, dx$ ✓

$\left[4x - \frac{2x^3}{3}\right]_{-\sqrt{2}}^{\sqrt{2}}$

$A = \left(4\sqrt{2} - \frac{4\sqrt{2}}{3}\right) - \left(-4\sqrt{2} + \frac{4\sqrt{2}}{3}\right)$ ✓

$= \frac{8\sqrt{2}}{3} - -\frac{8\sqrt{2}}{3}$

$= \frac{16\sqrt{2}}{3}$ units^2 ✓ (3)

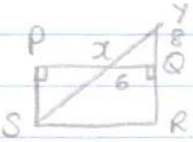
Award 1 for correct integral

Award 2 for substitution with correct answer

One mark deducted for not able to simplify $(\sqrt{2})^3$

Quite a number of students struggle with simplifying this expression.

d)



i) In $\triangle YQX$, $\triangle YRS$
$\angle Y$ is common ✓ no $\frac{1}{2}$m here.
$\angle YQX = \angle YRS = 90°$
(corresp $\angle$s $PQ \parallel SR$) 2

$\therefore \triangle YQX \;|||\; \triangle YRS$ (equiangular) ✓

Candidates need to be careful what is "given" in the question, $\angle XQY = 90°$ is not given. Parallel lines must be labelled using angles in parallel lines. Equiangular or all corresponding angles are equal can be used as a reason for similarity, not AAA.

No half mark awarded for this question.

ii) Let $QR = x \therefore SR = 3x$

$\frac{8+x}{3x} = \frac{8}{6}$ (same ratios in $|||\triangle$s)

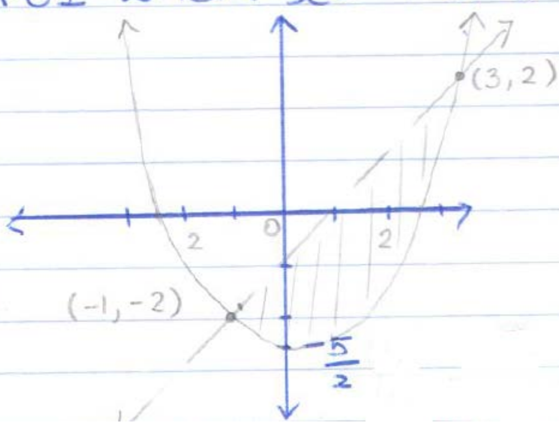
$6(8+x) = 24x$
$48 + 6x = 24x$
$48 = 18x$

$\therefore x = PS = 8/3 = 2\,2/3$ units ✓ 1

Generally well done. Try to answer as a fraction.

e) $\frac{x^2-5}{2} = x - 1$

$x^2 - 5 = 2x - 2$
$x^2 - 2x - 3 = 0$
$\therefore$ POI when $x = -1$ or 3



3

Most candidates did not show the point of intersection. This is part of the feature.

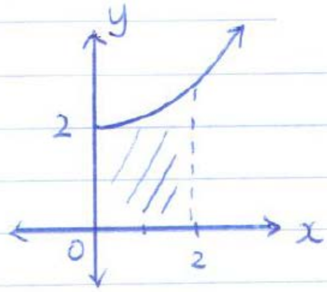
Award one for POI

Award one for correct parabola and linear function

Award one for the correct region and the correct lines

Minus half mark for each mistake.

f)



$V = \pi \int y^2\, dx$

$(e^x + e^{-x})^2 = e^{2x} + 2e^x e^{-x} + e^{-2x}$

$\therefore V = \pi \int_0^2 e^{2x} + e^{-2x} + 2\, dx$ ✓

$= \pi \left[\frac{e^{2x}}{2} - \frac{e^{-2x}}{2} + 2x \right]_0^2$ ✓

$= \pi \left[\left(\frac{e^4}{2} - \frac{e^{-4}}{2} + 4 \right) - \left(\frac{e^0}{2} - \frac{e^0}{2} + 0 \right) \right]$ 3

$= \pi \left(\frac{e^4 - e^{-4} + 8}{2} \right)$ units^3 ✓

Award 1 for the expansion. Almost half of the candidate experienced difficulty with the expansion

Award 1 for correct integral

Award 1 for simplified answer

g) $y' = e^{3x}$

$y = \frac{e^{3x}}{3} + c$

$x = 0, \quad y = 2$

$\therefore 2 = \frac{e^0}{3} + c$

$\therefore c = 5/3$

2

$y = \frac{e^{3x}}{3} + \frac{5}{3}$

$\therefore \quad y = \frac{e^{3x} + 5}{3}$

Award 1 for the correct value of c

Award 1 for the equation.

This is not a linear function, hence y = mx +b is incorrect.

Solutions to 2016 Assessment Task 2 2 unit.

8 a) (i) $f(x) = x + \frac{1}{x}$

$f(x) = -2$

So $x + \frac{1}{x} = -2$

$x^2 + 1 = -2x$

$x^2 + 2x + 1 = 0$ (1)

$(x+1)^2 = 0$

$x = -1$ (1)

Well done, but there were algebraic mistakes.

(ii) Assume odd.

$f(-x) = -f(x)$.

So $-f(x) = -\left(x + \frac{1}{x}\right)$

$f(-x) = -x + \frac{1}{-x}$

$= -x - \frac{1}{x} = -\left(x + \frac{1}{x}\right)$

odd function (1)

well done but many forgot the odd f~ rule

(iii) domain $x \neq 0$ (1) well done.

range $y \leq -2,\ y \geq 2$ (1)

prefer $f(x) \leq -2,\ f(x) \geq 2$.

Range was very poorly answered. Quite hard!

(b)

$$V = \pi \int_a^b y^2 \, dx$$

$$* \quad \doteq \frac{h}{3}\left[\left(y_0^2 + y_n^2\right) + 4\left(y_1^2 + y_3^2 + \cdots\right) + 2\left(y_2^2 + y_4^2 + \cdots\right)\right]$$

$h = \frac{b-a}{n}$, n = strips

$h = \frac{5-1}{4} = 1$

$y = \frac{1}{\sqrt{4+x^2}}$

	y_0	y_1	y_2	y_3	y_n
x	1	2	3	4	5
y	$\frac{1}{\sqrt{5}}$	$\frac{1}{\sqrt{8}}$	$\frac{1}{\sqrt{13}}$	$\frac{1}{\sqrt{20}}$	$\frac{1}{\sqrt{29}}$
y^2	$\frac{1}{5}$	$\frac{1}{8}$	$\frac{1}{13}$	$\frac{1}{20}$	$\frac{1}{29}$

(1)

$$V = \pi \int_1^5 y^2 \, dx \doteq \frac{1}{3}\left[\left(\frac{1}{5} + \frac{1}{29}\right) + 4\left(\frac{1}{8} + \frac{1}{20}\right) + 2\left(\frac{1}{13}\right)\right] \quad (1)$$

$$= \frac{4103}{11310}\pi \ u^3$$

$$\doteq 1.14 \ (2DP) \ u^3 \quad (1)$$

Very badly attempted. Look at the above formula, marked * and use it carefully.

8 (c) $\int_1^{e^3} \frac{7}{x}\,dx$

Well done by about 50% of students

$= 7\int_1^{e^3} \frac{1}{x}\,dx$

$= 7\ln x\Big]_1^{e^3}$ 1

note $\int \frac{1}{x}dx = \ln x$ etc.

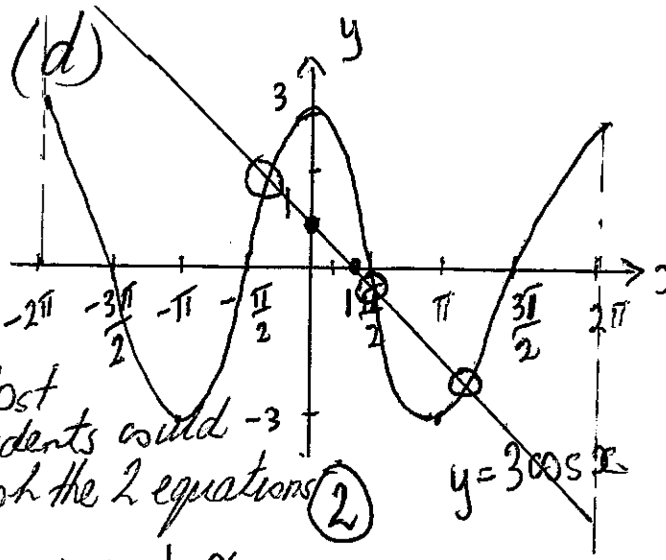
$= 7(\ln e^3 - \ln 1)$

$= 7(3\ln e - \ln 1)$

$= 7(3\times 1 - 0)$

$= 21$. 1

(d)



Most students could graph the 2 equations 2

now $y = 1 - x$.

$x = 0,\ y = 1 \quad (0,1)$

$y = 0 \quad x = 1^c \quad (1,0)$

since $\frac{\pi}{2} \doteqdot 1.6$, $x = 1^c$ is marked.

(d) (ii) $y = 3\cos x$

$y = 1 - x$

$3\cos x = 1 - x$.

has 3 solutions in $-2\pi \le x \le 2\pi$

Solutions are circled

1^{st} solution lies between $x = 0$ and $x = -\frac{\pi}{2} \doteqdot -1.6$

2^{nd} " " near $x = \frac{\pi}{2} \doteqdot 1.6$

3^{rd} " " between $x = \pi$ and $x = \frac{3\pi}{2}$

ie $x \doteqdot 3.1$ and $x \doteqdot 4.8$

So all the solutions must lie (could) between $x = -2$ and $x = 4$ 1

A very small number of students justifyed $-2 \le x \le 4$ in their own words.

(e) $x^2 + y^2 + 8x + 2y + 8 = 0$

$x^2 + 8x + 16 + y^2 + 2y + 1 = -8 + 16 + 1$

$(x+4)^2 + (y+1)^2 = 9$

$c(-4,-1) \quad r = 3$. 1

Concentric means same centre.

So

$(x+4)^2 + (y+1)^2 = r^2$

sub in $(1,7)$

$25 + 64 = r^2$ 1

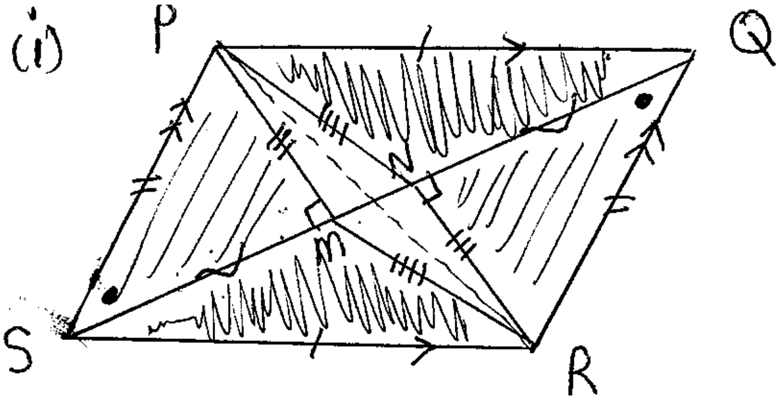
$\Rightarrow r = \sqrt{89}$

eq^n is

$(x+4)^2 + (y+1)^2 = 89$ 1

concentric $\Rightarrow$ same centre.
Question badly answered.

(f) (i)



PQRS is a parm $\Rightarrow$ $PQ = SR$, $PQ \parallel SR$ (1)
$PS = QR$, $PS \parallel QR$

(ii) In $\triangle PMS$, $\triangle RNQ$,
$PS = QR$ sides of parm
$P\hat{M}S = R\hat{N}Q = 90^\circ$ given
$P\hat{S}M = R\hat{Q}N$ alt. angles, transversal SQ.
$\therefore \triangle PMS \equiv \triangle RNQ$ (AAS) (1)
So $PM = RN$ matching sides.
$SM = QN$ matching sides

now in $\triangle PQN$, $\triangle RSM$
$SM = QN$ proved above.
$PQ = SR$ property of parm
$M\hat{S}R = \cancel{N}N\hat{Q}P$ opp angles in a parm are equal
So $\triangle PQN \equiv \triangle RSM$. (SAS)
$\therefore$ $PN = RM$. matching sides (1)
Hence PNRM is a parallelogram

Many students could not be bothered to write out a full proof. Hence items were assumed to be true without being proved first! Disappointing attempts!

Other discussions also looked positive eg using Pythagoras for $\triangle PMN$, $\triangle RNM$.

8 (g) $x = 3\sin(nt) + 6$ note 3 makes the graph bigger

6 shifts curve up 6 units.

$$P = \frac{2\pi}{n} = \frac{3\pi}{4}$$

$$2\pi = \frac{3\pi n}{4}$$

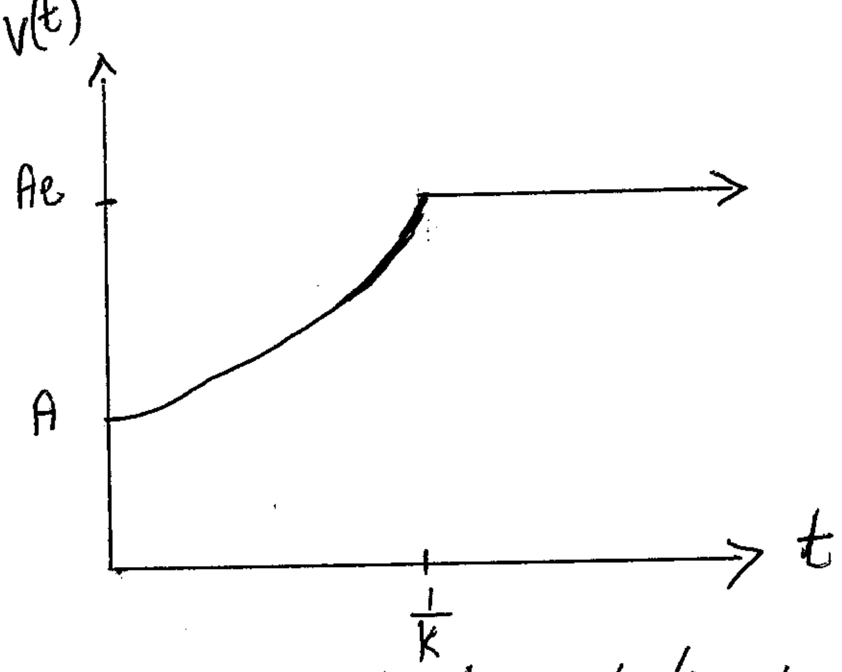
$$3\pi n = 8\pi$$

$$n = \frac{8\pi}{3\pi} = \frac{8}{3}$$

Question was very well answered when it was attempted.

1

(h)



2

Given A is a constant and $k > 1$

$V(t) = Ae$ for $t \geq \frac{1}{k}$

$V(t) = Ae^{kt}$ for $0 \leq t < \frac{1}{k}$

Badly attempted.
Most students left it out completely.
Those that did try, most were successful. But some did have a problem with the point $(\frac{1}{k}, Ae)$